Geometrical Approach to the Gauge Field Mass Problem. Possible Reasons for which the Higgs Bosons are Unobservable.

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The (4+d)-dimensional Einstein–Hilbert gravity action is considered in the Kaluza - Klein approach. The extra-dimensional manifold V_d is a Riemannian space with the d-parametric group of isometries G_d which acts on V_d by the left shifts and with an arbitrary non-degenerate left-invariant metric $\dot{g}_{ab}$. The gauge fields $\widehat{A}_\mu(x)$ are introduced as the affine connection coefficients of the fiber bundle with V_d being the fiber. The effective Lagrangian $L_{\rm eff}\{\widehat{A}_\mu(x), \dot{g}_{ab}\}$ is obtained as an invariant integral of the curvature scalar of the structure considered. The conditions on $\dot{g}_{ab}$ are formulated under which $L_{\rm eff}\{\widehat{A}_\mu(x), \dot{g}_{ab}\}$ contains in addition to the square of the gauge field strength tensor also the quadratic form of $\widehat{A}_\mu(x)$ and additional fields with pure gauge degrees of freedom. The eigenvalues of the quadratic form are calculated for the case of the gauge group $SO(3)$ and it is shown that they are not equal to zero in the case when $\dot{g}_{ab}$ is not proportional to the unit matrix.

As it is known, the most consistent approach to the theoretical description of the gauge fields is using Kaluza - Klein type theories of (4+d)-dimensional Einstein gravity ([1]-[3]). In this approach the properties of the gauge fields are the consequence of the geometrical and topological structure of the extra-dimensional manifold. The effective Lagrangian of this theory is obtained upon integration over the extra-dimensional manifold and, depending on the type of this space, can contain gauge fields, as well as various fermionic and scalar fields.

There is a well known difficulty in the description of massive gauge fields $\widehat{A}_\mu(x)$ which consists in the following. The quadratic form $\widehat{A}_\mu(x)\widehat{M}\widehat{A}_\mu(x)$ which describes mass terms in a 4-dimension Lagrangian is not invariant under inhomogeneous transformations of the gauge fields $\widehat{A}_\mu(x) \rightarrow \widehat{A}'_\mu(x) = S(x)\widehat{A}_\mu(x)S^{-1}(x) + S(x)\partial_\mu S^{-1}(x)$. As is well known, this problem is solved by the introduction into the theory of additional scalar fields with an appropriate transformation law and with such a self-interaction potential in the ordinary space-time which cause the scalar field to acquire a non-zero vacuum expectation value (VEV). The interaction of the gauge fields with this VEV produces mass terms for gauge fields. (the Higgs effect). This mechanism of the VEV generation should be necessarily accompanied by the quantum excitations over the VEV. The absence of the experimental observations of such excitations (Higgs bosons) impels one to think that it would be more natural to introduce into the theory of an intrinsic analog of the VEV object in the same manner as charges, lepton masses etc. are introduced. The origin of such objects should be regarded as the subject for a future study of the theory. In this work I would like to show that the metric of the extra-dimensional manifold in the Kaluza-Klein approach may be used as such an object.

We will shown that in multi-dimensional theories of the Kaluza-Klein type one can obtain the Lagrangian which is manifestly gauge invariant in multi-dimensional space-time and after the integration over the additional manifold reduces to an in effective 4-dimensionl Lagrangian which contains both the square of the strength tensor of gauge fields and the quadratic form of this fields. Note that a selection of parameters of the metric of V_d is possible such that corresponding additional fields are pure gauge degrees of freedom

and hence unobservable.

Let us briefly remind the basic structure of Kaluza - Klein type theories. The effective 4-dimensional action is obtained from these theories after the integration over the extra-dimensional manifold of the (4+d)-dimensional Einstein–Hilbert action:

$$S = \frac{1}{\kappa^2}\int d^{(4+d)}X\sqrt{-G}R = \int d^4x L_{\text{eff}}(x) \quad (1)$$

where $\kappa$is a (4+d)-dimensional gravitational constant, G, R are a metric and the scalar curvature in (4+d)-dimensional space. The coordinates X^A, $A = 0, 1, 2, 3, ..., 3+d$, are separated into the coordinates x^μ, $\mu = 0, ..., 3$, of the ordinary space–time, plus the coordinates y^α, $\alpha = 1, 2, ..., d$, of a compact d-dimensional manifold V_d.

We shall restrict our consideration to the Yang-Mills gauge fields with a r - parametric gauge group G_r, without taking into account effects of the gravitational field in ordinary space–time. In other words, we shall assume that (4+d)-dimensional space has a structure of the fiber bundle with the flat Minkowski space M_4 (with a metric $\gamma_{\mu\nu} = \text{diag}(1, -1, -1, -1)$) being its base and the fiber V_d being a Riemannian space with the r-parametric group of isometries G_r which acts transitively on V_d by the left shifts. This means that the dimension of V_d (i.e. d) is equal to r. Yang-Mills gauge fields are introduced as the affine connection coefficients

$$\widehat{A}_\mu(x) = A^a_\mu(x)E_a(y). \quad (2)$$

Here $E_a(y)$, $a = 1, ..., r$ are generators of the left shifts on V_d which obey the Lie algebra of G_r:

$$[E_a, E_b] = f^c_{ab}E_c, \quad (3)$$

where f^c_{ab} are the structure constants of G_r, and E_a have the form:

$$E_a(y) = \xi^\alpha_a(y)\partial/\partial y^\alpha, \quad (4)$$

$\xi^\alpha_a(y)$ are corresponding Killing vectors. Hereinafter it is convenient to use the variables y^α, A^a_μ, g_{ab}, $\gamma_{\mu\nu}$ and E_a in the dimensionless form. For this it is necessary to introduce a constant m^{-1} with the dimension of length.

Then in the covariant basis E_A, where

$$\left\{\begin{array}{ll} E_A = D_\mu \equiv \partial_\mu + meA^a_\mu E_a & A\ = 0, ..., 3 \\ E_A = mE_a & A\ = 3+a \end{array}\right. \quad (5)$$

the metric tensor G_{AB} of the (4+d)-dimensional manifold has the form:

$$G_{AB} = \begin{pmatrix} g_{ab} & 0 \\ 0 & \gamma_{\mu\nu} \end{pmatrix} \quad (6)$$

The commutation relations for the vector fields E_A are $[E_A, E_B] = C^D_{AB}E_D$, where

$$\left\{\begin{array}{ll} C^D_{AB} = eF^d_{\mu\nu} & \text{when } A = \mu, B = \nu, D = d; \\ C^D_{AB} = mf^d_{ab} & \text{when } A = a, B = b, D = d; \\ C^D_{AB} = 0 & \text{otherwise.} \end{array}\right. \quad (7)$$

Here
$F^a_{\mu\nu} = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) + emf^a_{bc}A^b_\mu(x)A^c_\nu(x)$ is the strength tensor of the gauge field, em is the gauge field coupling constant.

Using known formulae for the Christoffel symbols [4]

$$\begin{aligned} \Gamma^C_{AB} &= \frac{1}{2}G^{CD}(E_AG_{BD} + E_BG_{AD} - E_DG_{AB} \\ &- C^E_{AD}G_{BE} - C^E_{BD}G_{AE}) + \frac{1}{2}C^C_{AD}, \end{aligned} \quad (8)$$

and for the Riemann tensor

$$\begin{aligned} R^A{}_{BCD} &= C^E_{DC}\Gamma^A_{EB} - E_D\Gamma^A_{CB} + E_C\Gamma^A_{DB} \\ &- \Gamma^A_{DE}\Gamma^E_{CB} + \Gamma^A_{CE}\Gamma^E_{DB}, \end{aligned} \quad (9)$$

one can obtain the following expression for the scalar curvature [4]:

$$\begin{aligned} R = &- \frac{e^2}{4}g_{ab}\gamma^{\mu\rho}\gamma^{\nu\sigma}F^a_{\mu\nu}F^b_{\rho\sigma} \\ &+ R^{(b)} + R^{(f)} + R^{(M)} \end{aligned} \quad (10)$$

where $F^a_{\mu\nu}$ is defined in (7),

$$R^{(f)} = \frac{m^2}{2}(f^n_{am}f^m_{nb}g^{ab} + \frac{1}{2}f^n_{ad}f^m_{cb}g^{cd}g_{mn}g^{ab}) \quad (11)$$

is the scalar curvature of the fiber. $R^{(b)}=0$ in the case of the flat base,

$$R^{(M)}= -\frac{1}{2}\gamma^{\mu\nu}\Big\{g^{ab}(D_\mu(D_\nu g_{ab}))+D_\mu(g^{ab}(D_\nu g_{ab})) -\frac{1}{2}g^{bd}g^{ac}\big[(D_\mu g_{ad})(D_\nu g_{cb}) -(D_\mu g_{ac})(D_\nu g_{bd})\big]\Big\} \quad (12)$$

In order for when acting by the left shifts $E_a(y)$ on V_r the group G_r to be the isometry group the metric g on V_r must obey the Killing equation:

$$\xi_a^\sigma\partial_\sigma\widetilde{g}_{\alpha\beta}+\widetilde{g}_{\alpha\sigma}\partial_\beta\xi_a^\sigma+\widetilde{g}_{\beta\sigma}\partial_\alpha\xi_a^\sigma=0 \quad (13)$$

where $\widetilde{g}_{\alpha\beta}=g(\partial_\alpha,\partial_\beta)$ is the metric tensor in the coordinate basis, and ξ_a^α are the Killing vectors of the left shifts E_a defined in (4).

As it is known [5], the solution of (13) in the basis E_a has the form

$$g(E_a,E_b)=g_{ab}(y)=K_{aa_1}\dot{g}_{a_1a_2}\overline{K}_{b_1b}, \quad (14)$$

where $\dot{g}_{ab}=g(E_a^{(R)},E_b^{(R)})$ is the metric tensor in the left invariant basis of the right shifts $E_a^{(R)}=\eta_a^\alpha(y)\partial/\partial y_\alpha,\quad a=1,2,...,r;\quad \Big[E_a^{(R)},E_b^{(R)}\Big]=-f_{ab}^cE_c^{(R)},\quad \Big[E_a,E_b^{(R)}\Big]=0.$

$\dot{g}_{ab}$ is an arbitrary symmetrical non-degenerate $(r\times r)$ matrix with elements which do not depend on y_α. In eq. (14) $K_{ab}(y)=\xi_a^\alpha(y)\left(\eta_b^{-1}(y)\right)_\alpha$ is the matrix of the Ad-representation of G_r, $\overline{K}_{ab}=K_{ba}$ is the transposed matrix, and it is always possible to put det(K)=1.

The effective 4-dimensional Lagrangian has the form

$$L_{\text{eff}}(x)=\int_{V_r}d\mu(y)R \quad (15)$$

where the left-invariant measure is
$d\mu(y)=m^{-r}d^ry\sqrt{\widetilde{g}}=m^{-r}d^ry\sqrt{\dot{g}}/\det(\xi_a^\alpha)$.

$\widetilde{g}$ and $\dot{g}$ are the metric determinant in the coordinate and left-invariant basis, respectively.

It should be mentioned here that operators of the group G_r act on g_{ab} only by means of the generators E_a acting on $K_{ab}(y)$

$$(E_sK_{ab})=f_{sa}^rK_{rb}. \quad (16)$$

Therefore g_{ab} transforms under the action of the generators E_a as follows

$$E_cg_{ab}=f_{ca}^rg_{rb}+f_{cb}^rg_{ra} \quad (17)$$

and the inverse tensor g^{ab}, $\quad g^{ab}g_{ac}=\delta_c^a$ transforms as

$$E_cg^{ab}=-f_{cr}^ag^{rb}-f_{cr}^bg^{ra}. \quad (18)$$

It is easy to see that the expression (10) is manifestly invariant under the gauge transformations

$$\widehat{A}_\mu(x)\to\widehat{A}'_\mu(x)= S_{(E)}(x)\widehat{A}_\mu(x)S_{(E)}^{-1}(x)+S_{(E)}(x)(\partial_\mu S_{(E)}^{-1}(x)),$$
$$\dot{g}\to\dot{g}'=S_{(E)}(x)\dot{g}S_{(E)}^{-1}(x). \quad (19)$$

where $S_{(E)}(x)=\exp\left(\omega^a(x)E_a\right)$ is a local gauge transformation.

Since the metric tensor $g_{ab}(y)=K_{aa_1}\dot{g}_{a_1b_1}\overline{K}_{b_1b}$ is expressed in terms of the Ad-representation matrices K_{ab}, for further consideration it is convenient to pass to the matrix form of the generators E_s of the group G_r $E_s\to(f_s)_{ab}$, where $(f_s)_{ab}$ is a matrix form of the structure constants $f_{sa}^b=(f_s)_{ab}$.

Then the expression $(D_\mu g_{ab})$ may be rewritten as follows

$$(D_\mu g_{ab})= \partial_\mu\left(K(y)\dot{g}\overline{K}(y)\right)_{ab}+em(\widehat{A}_\mu)_{ac}(K(y)\dot{g}\overline{K}(y))_{cb} -em(K(y)\dot{g}\overline{K}(y))_{ac}(\widehat{A}_\mu)_{cb}, \quad (20)$$

where $(\widehat{A}_\mu)_{ab}=(A_\mu^sf_s)_{ab}$.

After substituting this expression into (10) - (15) and some algebra the effective Lagrangian may be written in the following form

$$L_{\text{eff}}\{A_\mu^s,\dot{g}_{ab}\}=L^{(F)}+L^{(M)}+L^{(f)}, \quad (21)$$

where

$$L^{(F)}= -\frac{e^2}{4\kappa^2}\int_{V_r}d\mu(y)\left(K(y)\dot{g}\overline{K}(y)\right)_{ab}\gamma^{\mu\rho}\gamma^{\nu\sigma}F_{\mu\nu}^aF_{\rho\sigma}^b \quad (22)$$

$$L^{(M)} = -\frac{1}{2\kappa^2}\int_{V_r} d\mu(y)$$
$$\Big\{(em)^2\big[Sp\{\widehat{A}_\mu\widehat{A}_\mu\}$$
$$- Sp\{\widehat{A}_\mu K(y)\dot{g}\overline{K}(y)\widehat{A}_\mu\overline{K}^{-1}(y)\dot{g}^{-1}K^{-1}(y)\}\big]$$
$$+em\big[Sp\{K(y)(\partial_\mu\dot{g})\dot{g}^{-1}K^{-1}(y)\widehat{A}_\mu\}$$
$$- Sp\{\widehat{A}_\mu\overline{K}^{-1}(y)\dot{g}^{-1}(\partial_\mu\dot{g})\overline{K}(y)\}\big]$$
$$+2Sp\{\dot{g}^{-1}(\partial^2_\mu\dot{g})\}+\frac{1}{2}[Sp\{\dot{g}^{-1}(\partial_\mu\dot{g})\}]^2$$
$$- \frac{3}{2}Sp\{\dot{g}^{-1}(\partial_\mu\dot{g})\dot{g}^{-1}(\partial_\mu\dot{g})\}\Big\}; \quad (23)$$

$$L^{(f)} = \frac{1}{\kappa^2}\int_{V_r} d\mu(y)R^{(f)}; \quad (24)$$

Here $\widehat{A}_\mu$ denotes the matrix $(A^s_\mu(x)f_s)_{ab}$; $\dot{g}$, $\dot{g}^{-1}$, K, $\overline{K}$, K^{-1}, $\overline{K}^{-1}$ denote corresponding matrices $\dot{g}_{ab}$, ... , $(\overline{K}^{-1})_{ab}$, and matrix multiplication is defined in the standard way.

The gauge transformation (19) now may be written in the usual matrix form
$(S(x))_{ab} = (exp(\omega^s(x)f_s))_{ab}$.

Let us take into account that the transformation
$K_{ab}(y) \rightarrow K'_{ab}(y) = (S(x)K(y)S^{-1}(x))_{ab} = K_{ab}(y')$ is equivalent to a change of the variables $y_\alpha \rightarrow y'_\alpha$ and that the measure $d\mu(y)$ is invariant under such transformations.

Then it is easy to see that eqs. (21) - (24) for the effective Lagrangian are gauge invariant

$$\widehat{A}_\mu(x) \rightarrow \widehat{A}'_\mu(x) = S(x)\widehat{A}_\mu(x)S^{-1}(x) + S(x)(\partial_\mu S^{-1}(x)),$$
$$\dot{g} \rightarrow \dot{g}' = S(x)\dot{g}S^{-1}(x). \quad (25)$$

The integration of (21) over $\int_{V_r} d\mu(y)$ in the general case of an arbitrary group G_r is a rather difficult prolbem, but the integration of the first term of $L_{\text{eff}}(x)$ in (21) is trivial when one uses known orthogonality relations for the irreducible unitary representations $K^\Lambda_{ab}(y)$

$$\int_{V_r} d\mu(y)K^{\Lambda_1}_{a_1b_1}(y)\overline{K}^{\Lambda_2}_{a_2b_2}(y) = \frac{v_r}{\dim(\Lambda_1)}\delta_{\Lambda_1\Lambda_2}\delta_{a_1a_2}\delta_{b_1b_2}. \quad (26)$$

Here the index Λ numbers different irreducible representations of the group G_r, $v_r = \int_{V_r} d\mu(y)$ is the volume of the manifold V_r and $\dim(\Lambda)$ is the dimension of K^Λ_{ab}.

Then we obtain

$$L^{(F)} = -\frac{v_r}{\kappa^2}\frac{e^2}{4r}Sp(\dot{g})F^a_{\mu\rho}F^a_{\nu\sigma}\gamma^{\mu\nu}\gamma^{\rho\sigma}.$$

One can see that in the case of arbitrary dependence of the matrix $\dot{g}_{ab}$ on x_μ, in addition to the gauge fields $A^a_\mu(x)$ the effective Lagrangian $L_{\text{eff}}\{A^s_\mu, \dot{g}_{ab}\}$ also contains Brans–Dicke type fields (because $Sp(\dot{g})$ depends on x_μ) and a set of scalar fields, with a complicated self-interaction potential, which interact with the gauge fields. In the general case these scalar fields belong to the different representations of G_r. The conditions can be formulated under which this set of fields form certain representations of G_r, and the Lagrangian of a standard Higgs type can be obtained. (See, for example, [7]).

But there is more interesting opportunity, the one to consider a left invariant metric $\dot{g}_{ab}$ as an intrinsic, independent of x_μ, characteristic of the gauge fields $A^a_\mu(x)$. More precisely, let us suppose that $\dot{g}_{ab}$ has the form $\dot{g}_{ab} = (S_0(x)\dot{\bar{g}}S_0^{-1}(x))_{ab}$, where $\dot{\bar{g}}_{ab}$ is a symmetrical matrix independent of x_μ and $S_0(x)$ is an arbitrary gauge transformation matrix.

It should be noted that this condition on $\dot{g}_{ab}$ is a direct generalization of the Einstein general relativity principle to high dimensions. Really, the Einstein general relativity principle is the statement that the 4-dimension metric tensor $\gamma^{\mu\rho}(x)$ which is associated to the arbitrary gravitational field, may be locally transformed to the flat Minkowski space tensor $\overline{\gamma}_{\mu\rho} = \text{diag}(1,-1,-1,-1)$ by the transformations of the gauge group. (In the case of gravity this is the group of general covariant transformations.)

The generalization of this principle to the fiber bundle structure under consideration is the statement that the left invariant metric tensor $\dot{g}_{ab}$ of any fiber V_r may be transformed by a gauge transformation $S_0(x)$ to a constant matrix $\dot{\bar{g}}$ independent of space-time coordinates x_μ.

It is easy to see that the gauge symmetry of

$L_{\rm eff}\{\widehat{A},\dot{g}_{ab}\}$ leads to the equality

$$L_{\rm eff}\{\widehat{A}_\mu, S_0(x)\dot{\bar{g}}_{ab}S_0^{-1}(x)\} = L_{\rm eff}\{\widehat{\overline{A}}_\mu, \dot{\bar{g}}_{ab}\} \quad (27)$$

where
$\overline{A}_\mu = S_0^{-1}(x)\widehat{A}_\mu S_0(x) - (em)^{-1}S_0^{-1}(x)(\partial_\mu S_0(x))$.

In other words, in this case the field $\dot{g}_{ab}$ has pure gauge degrees of freedom and there is a gauge condition $\partial_\mu(\dot{\bar{g}})_{ab} = 0$ when $L_{\rm eff}\{\widehat{A},\dot{g}_{ab}\}$ acquires the simplest form

$$L_{\rm eff}\{\overline{A}^a_\mu, \dot{\bar{g}}\} = -\frac{1}{4e_g^2}\widetilde{F}^a_{\mu\rho}\widetilde{F}^a_{\nu\sigma}\gamma^{\mu\nu}\gamma^{\rho\sigma} - m_0^2 B^s_\mu(x) M_{sr} B^r_\mu(x) + \Lambda(\dot{\bar{g}}). \quad (28)$$

Here

$$M_{sr} = -\frac{1}{v_r}\int_{V_r} d\mu(y) \Big[Sp\{(f_s)K(y)\dot{\bar{g}}\overline{K}(y)(f_r)\overline{K}^{-1}(y)\dot{\bar{g}}^{-1}K^{-1}(y)\} - Sp\{(f_s)(f_r)\}\Big]; \quad (29)$$

$$\Lambda(g) = \frac{1}{\kappa^2}\int_{V_r} d\mu(y) R^{(f)}; \quad (30)$$

and we have introduced new variables $B^s_\mu(x) = meS_0^{-1}(x)A^s_\mu(x)S_0(x) + S_0^{-1}(x)(\partial_\mu S_0(x))$ in order to get the canonical form of the gauge field strength tensor:

$$\widetilde{F}^a_{\mu\nu}(x) = \frac{1}{e_g}\left(\partial_\mu B^a_\nu - \partial_\nu B^a_\mu + f^a_{bc}B^b_\mu B^c_\nu\right).$$

In the (28)

$$e_g = \left(\frac{Sp(\dot{g})}{r}\frac{v_r}{m^2\kappa^2}\right)^{-1/2}; \quad m_0 = \left(\frac{v_r}{\kappa^2}\right)^{1/2}$$

We have not considered the gravitation sector of the theory, but it is well known [6] that in order to the gravitation sector, which arises as a result of deformations of the flat metric $\gamma_{\mu\nu}$ of the base, to coincides with ordinary Einstein gravity, one should put $v_r/\kappa^2 = 1/16\pi G_N$, where G_N is the Newton gravitational constant. Therefore it is necessary to put $m_0^2 = m_p^2/16\pi$, where m_p is the Plank mass. The parameter m^{-1}, which defines the length scale of the extra-dimension manifold, defines only the value e_g of the effective coupling constant of the gauge interaction. It is interesting that in the case of a simple group G_r the parameter e cancels out.

We are not able to develop here a general theory of the invariant integration over the Riemannian manifolds with an arbitrary group of isometries and shell restrict our further consideration to the case when V_r is a 3-dimensional Riemannian space with the isometry group of $G_r = SO(3)$.

It is convenient to use the parametrization of $SO(3)$ by the vectors y_α [8], where the vector $\vec{y}$ corresponds to the rotation around the axis $\vec{y}/y$ at the angle α: $y = tan(\alpha/2)$. In this parametrization we have $f^c_{ab} = \varepsilon_{abc}$,

the Killing vectors of the left shifts are

$$\xi_{a\alpha}(y) = \frac{1}{2}\left(\delta_{a\alpha} + y_a y_\alpha + \varepsilon_{a\alpha\beta}y_\beta\right) \quad (31)$$

and the matrix of the Ad-representation is

$$K_{ab}(y) = \frac{1}{1+y^2}\left[(1-y^2)\delta_{ab} + 2y_a y_b + 2\varepsilon_{abc}y_c\right]. \quad (32)$$

Finally, passing to the spherical coordinates in V_3 and taking into account that $\det(\xi) = \frac{1}{8}(1+{\rm y}^2)^2$, one writes the left-invariant measure in the form

$$\begin{aligned} d\mu(y) &= m^{-3}\sqrt{\dot{g}}\frac{1}{\det(\xi)}d^3y \quad (33)\\ &= 8m^{-3}\sqrt{\dot{g}}\frac{y^2}{(1+y^2)^2}dy\sin\Theta d\Theta d\varphi;\\ v_r &= \int_{V_r} d\mu(y) = 8\pi^2 m^{-3}\sqrt{\dot{g}}. \end{aligned}$$

The matrix $\dot{\bar{g}}_{ab}$ is an arbitrary non-degenerate (3×3) matrix. It may be diagonalized by a V rotation independent of x_μ and has the form

$$\dot{\bar{g}}_{ab} = V_{aa_1}g_{a_1}\delta_{a_1b_1}\overline{V}_{b_1b}$$

In the case of $G_r = SO(3)$ using the invariance of the measure $d\mu(y)$ and Jacobi identities for the structure constants f^c_{ab}, it is easy to see that the transformation $B^a_\mu \to V_{ab}B^b_\mu$ simultaneously diagonalizes also the quadratic form $B^a_\mu M_{ab} B^a_\mu$. After

the integration of (29), we obtain for the eigenvalues of the gauge field mass matrix:

$$M_{11} = \frac{1}{10}\Big[3\frac{(g_2-g_3)^2}{g_2g_3} + \frac{(g_1-g_3)^2}{g_1g_3} + \frac{(g_1-g_2)^2}{g_1g_2}\Big] \quad (34)$$

$$M_{22} = \frac{1}{10}\Big[3\frac{(g_1-g_3)^2}{g_1g_3} + \frac{(g_2-g_3)^2}{g_2g_3} + \frac{(g_1-g_2)^2}{g_1g_2}\Big] \quad (35)$$

$$M_{33} = \frac{1}{10}\Big[3\frac{(g_1-g_2)^2}{g_1g_2} + \frac{(g_3-g_1)^2}{g_3g_1} + \frac{(g_2-g_3)^2}{g_2g_3}\Big] \quad (36)$$

The integration of the fiber scalar curvature gives

$$\Lambda(\dot{g}) = -\frac{1}{2}\frac{m^2}{\kappa^2}v_r\left[2\mathrm{Sp}(\dot{\mathrm{g}}^{-1}) - \mathrm{Sp}(\dot{\mathrm{g}}^2)/\det(\dot{\mathrm{g}})\right] = \frac{m^2v_r}{2\kappa^2}\frac{1}{g_1g_2g_3}\Big[g_1^2+g_2^2+g_3^2 - 2(g_1g_2+g_1g_3+g_2g_3)\Big]. \quad (37)$$

One can see that for the case of compact V_3, (all g_i have identical signs) the eigenvalues of the gauge field mass matrix M_{sr} are positive definite. As we see, the values of mass in the considered approach are determined by the parameters $(g_i - g_j)$ which characterize the degree of deviation of the metric from the completely symmetrical case $g_1 = g_2 = g_3$. As it is well known, in this case V_3 is a symmetric space with the isometry group $SO(3) \times SO(3)$, and from (34-36) one can seen that $M_{ii} = 0$. If $g_1 = g_2 \neq g_3$ then the isometry group of V_3 is $SO(3) \times U(1)$ [9] and in this case $M_{11} = M_{22} \neq M_{33}$.

From (37) it is seen that cosmological constant $\Lambda = 0$ when $g_3 = (\sqrt{g_1} \pm \sqrt{g_2})^2$, and this condition does not impose serious restrictions on g_i.

In conclusion, in this paper we have shown how an effective Lagrangian for the massive gauge fields which does not contain any additional observable fields can be constructed. Such a Lagrangian can be rather simply written in the (4 + d)- dimensional form. (See expr. (21)—(24)). The integration of the corresponding expressions over extra dimensions in the general case leads to rather complicated formulae and it is convenient to perform this integration after imposing appropriate gauge conditions.

In this work the mass matrix of the gauge fields has been expressed in terms of the left-invariant metric of the additional space V_r and the calculations are made only with the purpose of illustrations of possible results for the simple case, when the gauge group is $SO(3)$. So we have not considered the problem of the determination of the metric parameters and the comparison of obtained expressions with experimental data. We did not touch also the problem, why the values $g_i - g_j$ are so small, that can reduce the Plank mass m_p down to values of gauge field masses observable experimentally. It would be interesting to consider these problems in the framework of ideas of the spontaneous compactification of multidimensional supergravity which were intensively studied by the D. V. Volkov group ([10]-[13]).

The author would like to thank I. Bandos and A.A. Zheltukhin for helpful discussions and interest to this work.

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